Radiación

Potenciales de Liénard-Wiechert

Radiación para partículas en movimiento.

$$\bar{E}(\bar{r},t) = \left.\frac{q(\hat{n}-\bar{\beta})}{\gamma^2(1-\bar{\beta}\cdot\hat{n})^3 R^2}\right|_{t_{ret}} + \left.\frac{q}{c}\frac{\hat{n}\times[(\hat{n}-\bar{\beta})\times\dot{\bar{\beta}}]}{(1-\bar{\beta}\cdot\hat{n})^3 R}\right|_{t_{ret}}$$

$$R(t') = |\bar{r}-\bar{r}_0(t')| \quad ; \quad \hat{n} = \frac{\bar{r}-\bar{r}_0(t')}{|\bar{r}-\bar{r}_0(t')|}$$

$\bar{r}_0(t') \longrightarrow$ Posición de la partícula en movimiento.

$$\bar{B}(\bar{r},t) = \hat{n}\Big|_{t_{ret}} \times \bar{E}$$

t_{ret} es el t' que cumple $\quad R(t') = c(t-t')$

Veamos Potencia:

$$dP = \bar{S}\cdot d\bar{S} = \bar{S}\cdot r^2 d\Omega\,\hat{r} \longrightarrow \frac{dP}{d\Omega} = \bar{S}\cdot r^2\hat{r}$$

Como quiero ver la energía transportada al infinito: Campo Lejano. $r \longrightarrow \infty$

$$\frac{dP}{d\Omega} = \frac{c}{4\pi} r^2 |\bar{E}_{rad}|^2$$

Aclaración

$$\boxed{\bar{E} = \bar{E}_v + \bar{E}_{rad}}$$

Ejercicio 2

Particula cargada con movimiento circular de frecuencia ω.

Calcular E, B, valor medio temporal de la distribucion de potencia e intensidad irradiada por ciclo.

$$\bar{r}_0(t') = a(\cos\omega t'\,\hat{x} + \text{Sen}\,\omega t'\,\hat{y})$$

$$\bar{v}(t') = a\omega(-\text{Sen}\,\omega t'\,\hat{x} + \cos\omega t'\,\hat{y})$$

$$\dot{\bar{v}}(t') = -a\omega^2(\cos\omega t'\,\hat{x} + \text{Sen}\,\omega t'\,\hat{y}) = -\omega^2\,\bar{r}_0(t')$$

$$\bar{\beta} = \frac{\bar{v}}{c}$$

Campo lejano $\quad \hat{n} = \frac{\bar{r} - \bar{r}_0(t')}{|\bar{r} - \bar{r}_0(t')|} \sim \hat{r}$

$$\hat{x} = \text{Sen}\,\theta\cos\varphi\,\hat{r} + \cos\theta\cos\varphi\,\hat{\theta} - \text{Sen}\,\varphi\,\hat{\varphi}$$

$$\hat{y} = \text{Sen}\,\theta\,\text{Sen}\,\varphi\,\hat{r} + \cos\theta\,\text{Sen}\,\varphi\,\hat{\theta} + \cos\varphi\,\hat{\varphi}$$

$$\bar{\beta} = \frac{a\omega}{c}\Big[(-\text{Sen}\,\omega t'\cos\varphi + \cos\omega t'\,\text{Sen}\,\varphi)\,\text{Sen}\,\theta\,\hat{r} + \dots$$

$$\dots + (-\text{Sen}\,\omega t'\cos\varphi + \cos\omega t'\,\text{Sen}\,\varphi)\cos\theta\,\hat{\theta} + \dots$$

$$\dots + (\text{Sen}\,\omega t'\,\text{Sen}\,\varphi + \cos\omega t'\cos\varphi)\,\hat{\varphi}\Big]$$

$$\bar{\beta} = \frac{a\omega}{c}\Big[-\text{Sen}(\omega t' - \varphi)\,\text{Sen}\,\theta\,\hat{r} - \text{Sen}(\omega t' - \varphi)\cos\theta\,\hat{\theta} + \cos(\omega t' - \varphi)\,\hat{\varphi}\Big]$$

$$\dot{\vec{\beta}} = -\frac{a\omega^2}{c}\left[\cos(\omega t'-\varphi)\operatorname{sen}\theta\,\hat{r} + \cos(\omega t'-\varphi)\cos\theta\,\hat{\theta} + \operatorname{sen}(\omega t'-\varphi)\,\hat{\varphi}\right]$$

Análisis de t_{ret}. Sale de resolver la sig. ecuación

$$|\bar{r}-\bar{r}_0(t')| = c(t-t')$$

$$\sqrt{(\bar{r}-\bar{r}_0)\cdot(\bar{r}-\bar{r}_0)} = c(t-t')$$

$$\sqrt{r^2+r_0^2-2\bar{r}\cdot\bar{r}_0} = c(t-t')$$

$$r\sqrt{1+\underbrace{\cancel{\left(\frac{r_0}{r}\right)^2}}_{O\left(\left(\frac{r_0}{r}\right)^2\right)} - 2\frac{\bar{r}\cdot\bar{r}_0}{r^2}} = c(t-t')$$

$\boxed{\begin{array}{l}\underline{\text{Campo lejano}}\\ \frac{r_0}{r} \ll 1\end{array}}$

Hago taylor $\sqrt{1-2x} \simeq 1-x$ $\quad x \ll 1$

$$r\left(1-\frac{\bar{r}\cdot\bar{r}_0}{r^2}\right) = c(t-t')$$

$$t' \simeq t - \frac{r}{c} + \frac{\bar{r}\cdot\bar{r}_0}{cr}$$

Como voy a evaluar en funciones armónicas.

$$\omega t' \simeq \omega t - \frac{\omega r}{c} + \underbrace{\frac{\omega}{c}\frac{\bar{r}\cdot\bar{r}_0}{r}}_{O\left(\frac{\omega}{c}r_0\right) = O\left(\frac{\omega a}{c}\right)}$$

Como mi mov. es circular $|\bar{v}| = \omega a$

$$O\left(\frac{v}{c}\right) = O(\beta)$$

Si al evaluar en t_{ret} uso $t_{ret} = t - \frac{r}{c}$

estoy tirando una corrección de orden β

estoy en régimen $\beta \ll 1 \Rightarrow$ No relativista.

Voy a usar esta aproximación no relativista.

$$1 - \bar{\beta}\cdot\hat{n} \simeq 1 \quad ; \quad \underbrace{\hat{n} - \bar{\beta} \simeq \hat{n}}_{\text{No Rel.}} \underbrace{\simeq \hat{r}}_{\text{Campo Lejano.}}$$

$$\bar{E}_{rad}(\bar{r}, t) = \frac{q}{c} \frac{\hat{r} \times [\hat{r} \times \dot{\bar{\beta}}]}{r} \Bigg|_{t' = t - \frac{r}{c}}$$

$$\hat{r} \times (\hat{r} \times \dot{\bar{\beta}}) = \frac{a\omega^2}{c}\left(\cos(\omega t' - \varphi)\cos\theta\,\hat{\theta} + \operatorname{sen}(\omega t' - \varphi)\hat{\varphi}\right)$$

$$\boxed{\bar{E}_{rad}(\bar{r}, t) = \frac{q a \omega^2}{c^2 r}\left[\cos(\omega t - \Phi)\cos\theta\,\hat{\theta} + \operatorname{sen}(\omega t - \Phi)\hat{\varphi}\right]}$$

$$\Phi = \frac{\omega r}{c} + \varphi$$

$$\frac{dP(t)}{d\Omega} = \frac{c}{4\pi} r^2 |\vec{E}_{rad}|^2 = \frac{c}{4\pi} \frac{q^2 a^2 \omega^4}{c^4} \left(\cos^2(\omega t - \Phi) \cos^2\theta + \operatorname{sen}^2(\omega t - \Phi) \right)$$

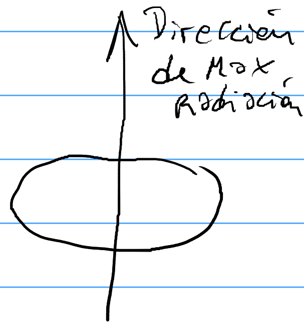
$$= \frac{c}{4\pi} \frac{q^2 a^2 \omega^4}{c^4} \left(1 - \cos^2(\omega t - \Phi) \operatorname{sen}^2\theta \right)$$

$$\left\langle \frac{dP(t)}{d\Omega} \right\rangle = \frac{1}{\tau} \int_0^{\tau} \frac{dP(t)}{d\Omega} dt \qquad \text{con } \tau = \frac{2\pi}{\omega}$$

$$\langle \cos^2(\omega t - \Phi) \rangle = \frac{1}{2}$$

$$\boxed{\left\langle \frac{dP(t)}{d\Omega} \right\rangle} = \frac{q^2 a^2 \omega^4}{4\pi c^3} \left(1 - \frac{1}{2} \operatorname{sen}^2\theta \right) =$$

$$\boxed{= \frac{q^2 a^2 \omega^4}{4\pi c^3} \frac{1}{2} (1 + \cos^2\theta)}$$



Potencia total irradiada

$$P = \iint_{\Omega} \left\langle \frac{dP}{d\Omega} \right\rangle d\Omega = \frac{q^2 a^2 \omega^4}{8\pi c^3} \int_0^{2\pi}\int_0^{\pi} (1 + \cos^2\theta) \operatorname{sen}\theta \, d\theta \, d\varphi$$

$$\begin{cases} \int_0^{\pi} \operatorname{sen}\theta \, d\theta = (-\cos\theta)\Big|_0^{\pi} = 2 \\ \int_0^{\pi} \cos^2\theta \operatorname{sen}\theta \, d\theta = \int_{-1}^{1} u^2 du = \frac{2}{3} \end{cases}$$

$\cos\theta = u$

$-\operatorname{sen}\theta \, d\theta = du$

$$\boxed{P} = \frac{q^2 a^2 \omega^4}{4c^3}\left(2+\frac{2}{3}\right) = \boxed{\frac{2}{3}\frac{q^2 a^2 \omega^4}{c^3}}$$

Pot. de Larmor no rel. : $P = \frac{2}{3}\frac{q^2}{c^3}|\dot{\bar{v}}|^2$

Ejercicio 13

Tengo particula oscilando en el eje z $\quad z(t') = a \cos \omega_0 t'$

a) Llegar a formula para distribución de Potencia instantánea irradiada.

$$\frac{dP}{d\Omega}(t') = \frac{q^2 c \beta^4}{4\pi a}\frac{\cos^2 \omega_0 t' \operatorname{sen}^2\theta}{(1+\beta \operatorname{sen}\omega_0 t' \cos\theta)^5} \; ; \; \beta = \frac{a\omega_0}{c}$$

Solucion:

$$\bar{r}_0(t') = a \cos \omega_0 t' \, \hat{z}$$

$$\bar{v}(t') = -a\omega_0 \operatorname{sen}\omega_0 t' \, \hat{z}$$

$$\dot{\bar{v}}(t') = -a\omega_0^2 \cos \omega_0 t' \, \hat{z}$$

Campo lejano $\hat{n} \simeq \hat{r} \qquad R \simeq r$

$$\frac{dP}{d\Omega}(t) = \frac{dE}{dt\, d\Omega} = \frac{dE}{dt'\, d\Omega}\cdot\frac{dt'}{dt} = \overbrace{\frac{dP(t')}{d\Omega}}^{\text{quiero.}}\cdot\frac{dt'}{dt}$$

$$\frac{dP}{d\Omega}(t') = \underbrace{\frac{c}{4\pi} r^2 |\bar{E}_{rad}|^2}_{t'} \underbrace{(1-\bar{\beta}\cdot\hat{n})}_{\frac{dt}{dt'}} \longrightarrow$$ Sale de la ec. implícita de t_{ret}.

$$|\bar{E}_{rad}|^2 = \frac{q^2}{c^2} \frac{|\hat{r}\times[(\hat{r}-\bar{\beta})\times\dot{\bar{\beta}}]|^2}{(1-\hat{\beta}\cdot\hat{r})^6\, r^2}$$

$$\hat{z} = \cos\theta\, \hat{r} - \operatorname{Sen}\theta\, \hat{\theta}$$

$$\bar{\beta}\times\dot{\bar{\beta}} = 0 \quad \text{porque} \quad \bar{\beta} \,//\, \dot{\bar{\beta}}$$

$$\hat{r}\times\dot{\bar{\beta}} = \hat{r}\times\left(-\frac{a\omega_0^2}{c}\cos\omega_0 t' \left[\cos\theta\,\hat{r} - \operatorname{Sen}\theta\,\hat{\theta}\right]\right) =$$

$$= \frac{a\omega_0^2}{c}\cos\omega_0 t' \operatorname{Sen}\theta\, \hat{\varphi}$$

$$|\hat{r}\times[(\hat{r}-\bar{\beta})\times\dot{\bar{\beta}}]|^2 = \frac{a^2\omega_0^4}{c^2}\cos^2\omega_0 t' \operatorname{Sen}^2\theta$$

$$1-\bar{\beta}\cdot\hat{r} = 1+\frac{a\omega_0}{c}\operatorname{Sen}\omega_0 t' \cos\theta$$

$$|\bar{E}_{rad}|^2 = \frac{q^2}{c^2}\cdot\frac{\frac{a^2\omega_0^4\cos^2\omega_0 t' \operatorname{Sen}^2\theta}{c^2}}{\left(1+\frac{a\omega_0}{c}\operatorname{Sen}\omega_0 t'\cos\theta\right)^6 r^2}$$

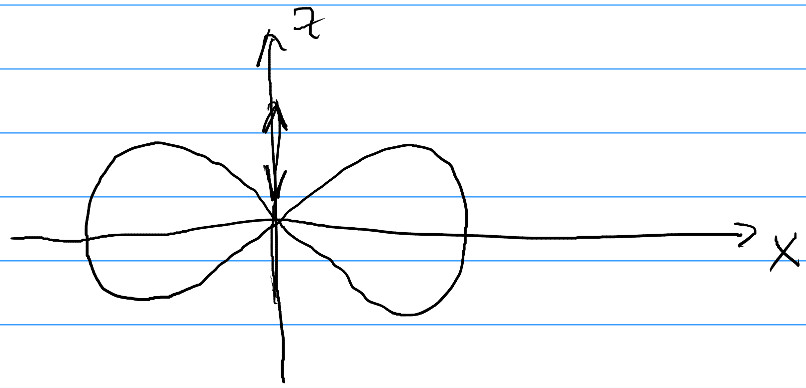
$$\frac{dP}{d\Omega}(t') = \frac{c}{4\pi}\frac{q^2}{c^2}\frac{\frac{a^2\omega_0^4}{c^2}\cos^2\omega_0 t' \operatorname{sen}^2\theta}{\left(1+\frac{a\omega_0}{c}\operatorname{sen}\omega_0 t' \cos\theta\right)^5}$$

$$\beta \equiv \frac{a\omega_0}{c}$$

$$\boxed{\frac{dP}{d\Omega}(t') = \frac{q^2 c\beta^4}{4\pi a^2}\frac{\cos^2\omega_0 t' \operatorname{sen}^2\theta}{(1+\beta\operatorname{sen}\omega_0 t'\cos\theta)^5}}$$

Caso no relativista $\beta \ll 1$

$$\frac{dP}{d\Omega}(t') = \frac{q^2 c\beta^4}{4\pi a^2}\cos^2\omega_0 t' \operatorname{sen}^2\theta$$



Caso Relativista

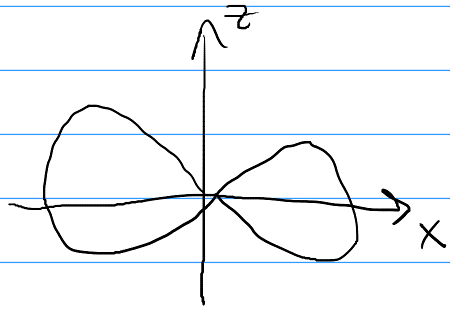
$$\frac{dP}{d\Omega}(t') = \frac{q^2 c\beta^4}{4\pi a^2}\cos^2\omega_0 t' \frac{(1-\cos^2\theta)}{(1+\beta\operatorname{sen}\omega_0 t'\cos\theta)^5}$$

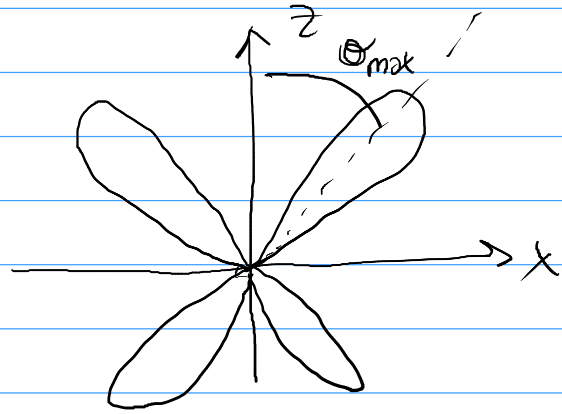
$x = \cos\theta$ $\qquad$ $\tilde{\beta} = \beta\operatorname{sen}\omega_0 t'$

$$f(x) = \frac{1-x^2}{(1+\tilde{\beta}x)^5} \xrightarrow{\text{Máx}} \quad x_{max} = \cos\theta_{max} = \frac{1-\sqrt{1+15\tilde{\beta}}}{3\tilde{\beta}}$$

$$\tilde{\beta} \xrightarrow[\lim.]{} \pm 1 \qquad \cos\theta_{max} \longrightarrow \pm 1 \;;\; \theta_{max} \begin{cases} \to 0 \\ \to \pi \end{cases}$$

Cada vez
mas relativista





Ejercicio 11

Particula no relativista choca con un potencial central V(r) (repulsivo).

mostrar que la formula para la energia total irradiada es

$$\Delta W = \frac{4}{3}\frac{z^2 e^2}{m^2 c^3}\sqrt{\frac{m}{2}} \int_{r_{min}}^{\infty} \left|\frac{dV}{dr}\right|^2 \frac{dr}{\sqrt{V(r_{min}) - V(r)}}$$

Solución

$$\Delta W = \int_{-\infty}^{\infty} P(t)\,dt \underset{\text{Pq es no relativista}}{=} \int_{-\infty}^{\infty} P(t')\,dt'$$

Formula de larmor $\quad P(t') = \frac{2}{3}\frac{q^2}{c^3}|\ddot{\vec{r}}|^2 =$

Recuerdo que como el problema es de potencial $\quad = \frac{2}{3}\frac{q^2}{c^3}|\ddot{r}_0|^2$

central, la partícula va a acercarse hasta cierto radio y va a salir disparada en otra dirección

$t'=0$ en $r_{0_{min}}$ al que se acerca.

$$\Delta \omega = \int_{-\infty}^{\infty} P(t')dt' = 2\int_{0}^{\infty} P(t')\,dt'$$

Cambio de variable $\quad r_0(t') = r$

$\dot{r}_0(t')\,dt = dr$

$$\Delta \omega = 2\int_{r_{min}}^{\infty} P(r)\cdot\frac{dr}{\dot{r}}$$

$$P = \frac{2}{3}\frac{q^2}{c^3}\ddot{r}^2 \longrightarrow \ddot{r}(r)$$

↳ Newton

$$\ddot{r} = -\frac{1}{m}\frac{dV}{dr} \longleftarrow m\ddot{r} = -\frac{dV}{dr}$$

Supongo que la energía irradiada es mínima

$$\Rightarrow \underline{E} = \text{Se conserva} \rightarrow E = \frac{m\dot{r}^2}{2} + V(r)$$

$$V(r_{min}) = E \Rightarrow V(r_{min}) = \frac{m\dot{r}^2}{2} + V(r)$$

Por punto de retorno

$$\dot{r} = \sqrt{\frac{2}{m}} \sqrt{V(r_{min}) - V(r)}$$