Instantaneous Velocity Using Photogate Timers

Article *in* The Physics Teacher · April 2010

DOI: 10.1119/1.3361999

John Wolbeck
State University of New York

Instantaneous Velocity Using Photogate Timers

John Wolbeck, SUNY Orange Community College, Middletown, NY

Photogate timers are commonly used in physics laboratories to determine the velocity of a passing object. In this application a card attached to a moving object breaks the beam of the photogate timer providing the time for the card to pass. The length L of the passing card can then be divided by this time to yield the average velocity (or speed) of the passing object:

$$V_{avg} = L/t. \tag{1}$$

The problem

The equation $V = L/t$ looks simple enough, but a discrepancy in our data began to show up when the photogate timer was placed near the beginning of an accelerating object's run. We could guess that the instantaneous velocity occurs as the center of the card passes through the gate, but since distance and time are not linearly related for an accelerating system (think $d = ½\, at^2$), this may not be true.[1] In fact if the photogate were placed right at the leading edge of the card, the instantaneous velocity would occur when only the first one-fourth of the card had entered the gate.

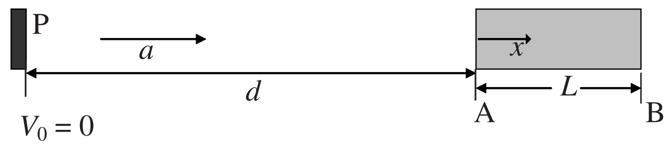
The exact location of this instantaneous velocity center is important. For example, when doing conservation of energy experiments, the potential energy drop height must be measured from the center of the card at $V = 0$ to the center of the card at the exact moment that the instantaneous velocity center on the card is centered in the photogate. In another common application, two gates can be used to calculate acceleration of a passing object by finding the velocity at each gate and using the "velocity squared" equation:

$$V_f^2 - V_0^2 = 2ad. \qquad (2)^{2,3}$$

However, the distance between the two instantaneous velocity measurements must actually be the distance between the centers of velocity when the card first reaches each gate and not simply the distance between the gates.[4] If we ignore this fact, the distance error could be as large as one-fourth of a card length if the first gate is close to the starting position.

The model

In actual use, the object usually accelerates toward a fixed photogate. In order to simplify the equations, we will consider the analogous case where a photogate accelerates past the object. With a card at rest, we will uniformly accelerate a photogate timer, P, from rest over a distance d and past a fixed card of length L. This is simply the experiment from the point of view of the card.



With $V_0 = 0$ the velocity squared equation from kinematics ($V_f^2 - V_0^2 = 2a\Delta d$) [Eq. (2)] lets us predict the velocity of our photogate at the front (A) and back (B) of the card as:

Velocity of the photogate at point A: $V_A = \sqrt{2ad}$. (3)

Velocity of the photogate at point B: $V_B = \sqrt{2a(d+L)}$. (4)

For a uniformly accelerating system: $V_{avg} = (V_A + V_B)/2$ (5)

We can now use Eq. (2) to find x, the distance along the card where the average velocity, V_{avg}, actually occurs:

$$V_{avg}^2 - V_A^2 = 2ax. \tag{6}$$

Combining Eqs. (3) and (4) into Eq. (5) for V_{avg}, we get

$$\left(\frac{\sqrt{2ad} + \sqrt{2a(d+L)}}{2}\right)^2 - \left(\sqrt{2ad}\right)^2 = 2ax. \tag{7}$$

FOILing and dividing by $2a$ yields

$$d + 2\sqrt{d}\sqrt{d+L} + d + L - 4d = 4x. \tag{8}$$

(Note that for $d = 0$ we find that $x = L/4$).

Continuing the solution yields

$$x = \frac{1}{4}\left[2d\left(\sqrt{1+\frac{L}{d}} - 1\right) + L\right]. \tag{9}$$

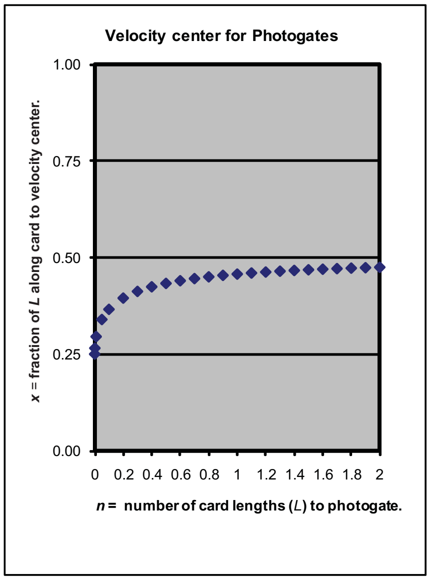
The solution

Equation (8) clearly shows that the limit at $d = 0$ yields $x = L/4$. If we now let d go to infinity, the other limit is a bit harder to find. Looking at Eq. (9), at infinity the L/d term goes to zero and the $(\sqrt{1}-1)$ is seemingly going to zero, yielding $¼L$ again. However, in the limit, $2d$ is going to infinity while the $(\sqrt{1}-1)$ is going to zero, so the solution is undefined by this method. We could use L'Hopital's rule and take the limit of the derivatives of Eq. (9) as d goes to infinity and we will get the correct solution of $½L$.

An interesting alternative to L'Hopital's rule is to create a graph of the equation. By letting $d = nL$, we can rewrite Eq. (9) as a function where n equals the number of card lengths measured from the starting position to the photogate:

$$x = \frac{L}{4}\left[2n\left(\sqrt{1+\frac{1}{n}}-1\right)+1\right]. \qquad (10)$$

n	*x*
1E-08	0.250
0.001	0.265
0.01	0.295
0.05	0.340
0.1	0.366
0.2	0.395
0.3	0.412
0.4	0.424
0.5	0.433
0.6	0.440
0.7	0.445
0.8	0.450
0.9	0.454
1	0.457
2	0.475
3	0.482
4	0.486
5	0.489
10	0.494
20	0.497
40	0.498
50	0.499
100	0.499
200	0.500



This graph depicts the location of the velocity center (x) for a card that is dropped from less than two card lengths ($2n$) above a photogate.

The table and graph, based on Eq. (10), nicely show that at $n = 0$, $x = ¼L$, and that as n gets larger, x moves to the center of the card at $x = ½L$. In fact, the location of the instantaneous velocity closes in on $½L$ surprisingly quickly. For example, when the object starts at only one-half of a card length away from the photogate, the velocity center is already at $0.43L$. Unfortunately, this represents a 14% difference from $½L$ and may not be an acceptable level of error for most experiments.

Testing the model

An experiment I use in the classroom to verify this relationship incorporates the use of a 30-cm long card that is dropped though a photogate timer. By dropping the card from different heights (n) above the photogate, conservation of energy can be used to predict the location of the velocity center on the card. These data can be graphed and compared with the theoretical solution given in Eq. (10).

Conclusions

Can we now assume that the center of velocity is at the center of the card as it passes a photogate? The answer depends on the distance to the photogate and the amount of error that you are willing to tolerate. In general, if the photogate is more than five card lengths away from the start of a run, the velocity center will be within 2.3% of the center of the card. If the photogate is closer than two card lengths, the amount of error climbs above 5%. One quick fix is to use narrow cards since this will increase the distance ratio to the gates, yielding more accurate results. A more scientific fix is to use the table generated above and to have students mark the actual velocity center on the card as it reaches each gate. This requires more effort and understanding on the student's part, but we are working with timers that can measure events to within thousandths of a second so the effort may be justified.

References

1. P. F. Hinrichsen, *AAPT Announcer* **13**(4), 101, GG6 (1983).
2. PASCO scientific, 10101 Foothills Blvd., Roseville, CA 95747-7100; www.pasco.com.
3. Giancolli, *Physics* (Pearson Education Inc., Upper Saddle River, NJ, 2005).
4. P. F. Hinrichsen, *Apparatus for Teaching Physics* (AAPT, College Park, MD, 1998). [AU:can you provide chapter title information or page numbers?]

John Wolbeck *is a licensed professional engineer and a professor of engineering at SUNY Orange Community College, where he has taught physics and engineering courses for the past 15 years. He is currently the interim chair for the Department of Science, Engineering, and Architecture.*

SUNY Orange Community College, 115 South St., Middletown, NY 10940; John.Wolbeck@Sunyorange.edu