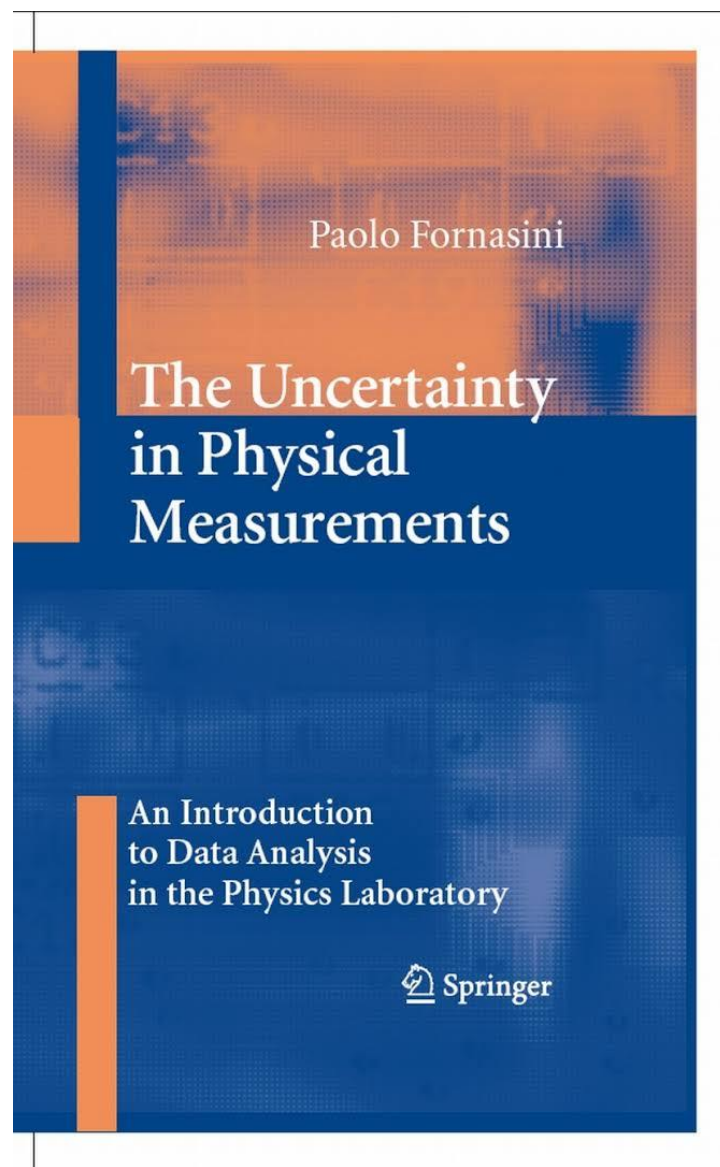


LABORATORIO 4

TRATAMIENTO DE DATOS

Adriana Márquez
amarquez@df.uba.ar



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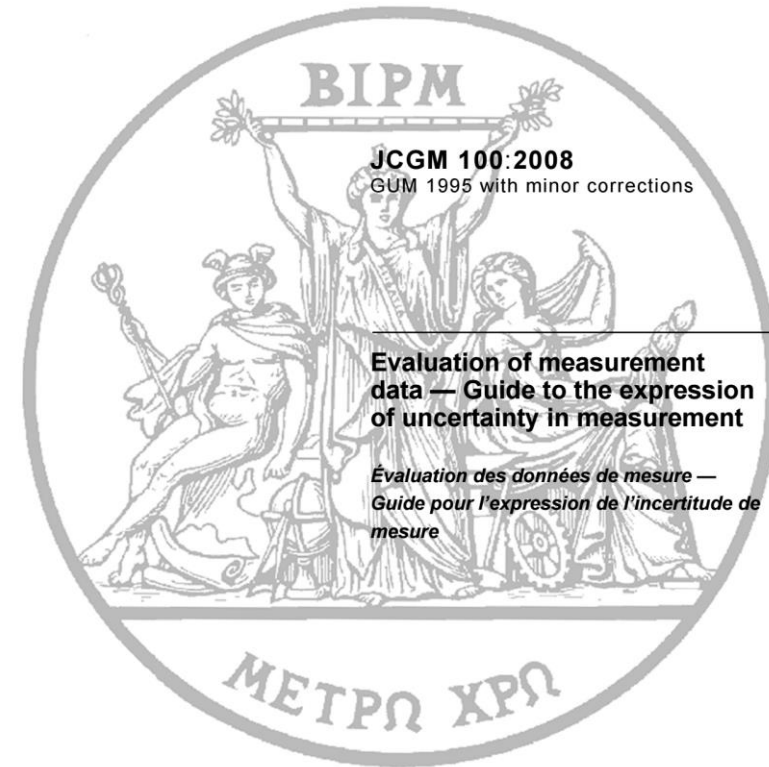
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GUM (*Guide to the Expression of Uncertainty in Measurement*)

Guía para evaluar y expresar la incertidumbre en las mediciones científicas e industriales

JCGM (*Joint Committee for Guides in Metrology*)

Comité conjunto de organizaciones mundiales vinculadas a la “standarización” y la metrología que crea y actualiza la GUM y el Vocabulario internacional de metrología (VIM)



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<https://www.bipm.org/en/committees/jc/jcgm/publications>

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Un poco de historia...

- 1977 Comité International des Poids et Mesures (CIPM), solicita al Bureau International des Poids et Mesures (BIPM) elaborar un consenso internacional sobre cómo expresar las incertidumbres en las mediciones
- 1980 11 laboratorios internacionales de Patrones (Standards) elaboran *Recommendation INC-1, Expression of Experimental Uncertainties*
- 1993 Se crea el JCGM (*Joint Committee for Guides in Metrology*)
- 2017 Se compilan los documentos bajo la denominación de GUM (*Guide to the Expression of Uncertainty in Measurement*)



First edition 2023

- *Ofrece un racional para evaluar, expresar y utilizar la incertidumbre de la medición.*
- *Proporciona una base para la comparación internacional de los resultados de las mediciones*
- *Para su aplicación en la ciencia, el comercio, la salud, la seguridad y otras actividades sociales.*

Annex A Overview of the parts of the GUM

Table A.1: Overview of the parts (published and planned) of the Guide to the expression of uncertainty in measurement

GUM part	Old reference	Title
JCGM GUM-1 ^a	JCGM 104	Guide to the expression of uncertainty in measurement — Part 1: Introduction
JCGM GUM-2 ^b		Guide to the expression of uncertainty in measurement — Part 2: Concepts
JCGM GUM-3 ^c	JCGM 100	Guide to the expression of uncertainty in measurement — Part 3: GUM:1995 with minor corrections
JCGM GUM-4 ^d	JCGM 106	Guide to the expression of uncertainty in measurement — Part 4: The role of measurement uncertainty in conformity assessment
JCGM GUM-5 ^b	JCGM 110 ^h	Guide to the expression of uncertainty in measurement — Part 5: Examples
JCGM GUM-6 ^e	JCGM 103 ^h	Guide to the expression of uncertainty in measurement — Part 6: Developing and using measurement models
JCGM GUM-7 ^f	JCGM 101	Guide to the expression of uncertainty in measurement — Part 7: Propagation of distributions using a Monte Carlo method
JCGM GUM-8 ^g	JCGM 102	Guide to the expression of uncertainty in measurement — Part 8: Extension to any number of output quantities
JCGM GUM-9 ^b		Guide to the expression of uncertainty in measurement — Part 9: Statistical models and data analysis for interlaboratory studies
JCGM GUM-10 ^b		Guide to the expression of uncertainty in measurement — Part 10: Applications of the least squares method
JCGM GUM-11 ^b		Guide to the expression of uncertainty in measurement — Part 11: Bayesian methods
JCGM GUM-12 ^b		Guide to the expression of uncertainty in measurement — Part 12: Basic methods for uncertainty propagation

^a JCGM 104:2009 was replaced by this document

^b Planned

^c Available as JCGM 100:2008 [7]

^d Available as JCGM 106:2012 [10]

^e Available as JCGM GUM-6:2020 [11]

^f Available as JCGM 101:2008 [8]

^g Available as JCGM 102:2011 [9]

^h Reference of the committee draft

Presentaciones

Fecha	Tema	Material de referencia	Grupo
2/9	Cantidades físicas – ¿Qué es medir?	Cap. 1	
9/9	Unidades de medida	Cap. 2	
16/9	Instrumentos de medición	Cap. 3	
23/9	Incertidumbre en mediciones directas	Cap. 4 - JCGM 100:2008 Parte 4 /JCGM GUM 1-2023	G1
7/10	Incertidumbre en mediciones indirectas	Cap. 8 - JCGM 100:2008 Parte 5 /JCGM GUM 1-2023	G2
14/10	Nivel de confianza	Cap. 9 - JCGM 100:2008 Parte 6 y 7 /JCGM GUM 1-2023	G3
14/10	Correlación de cantidades físicas- Método Máxima Verosimilitud – Cuadrados mínimos	Cap. 10	G4
14/10	Cuadrados mínimos con incertidumbre en ambas variables Ajuste por regresión de distancia ortogonal (ODR) -OLS	link1 , link2 , link3 [hasta 2:40]	G5
14/10	Coeficiente de correlación y coeficiente de determinación. Bandas de confianza y bandas de predicción	Link1 , Link2 , Link3 , link 4 , link 5	G6
14/10	Test de Chi Cuadrado	Cap. 11	G7
14/10	Tratamiento de señales: filtros en el espacio directo: media móvil y filtrado de Savitzky-Golay	link1 , link2 , savitzky1964	G8

Lectura individual
conceptos de
estadística Cap. 5 a
Cap. 7