Distribución Binomial: Esperanza y Varianza

$$(p+q)^n = \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k}$$

① $$n\,(p+q)^{n-1} = \sum_k \binom{n}{k} k\, p^{k-1} q^{n-k} \qquad n\,(n-1)\,(p+q)^{n-2} = \sum_k \binom{n}{k} k(k-1)\, p^{k-2} q^{n-k}$$

② $$n\,(p+q)^{n-1}\,p = \sum_k \binom{n}{k} k\, p^{k} q^{n-k} \qquad n\,(n-1)\,(p+q)^{n-2}\,p^2 = \sum_k \binom{n}{k} (k^2-k)\, p^{k} q^{n-k}$$

③ $$n\,p = \langle k \rangle \qquad n\,(n-1)\,p^2 = \langle k^2 \rangle - \langle\, k\, \rangle$$

$$E(X) = \langle k \rangle = np \qquad Var(X) = \langle k^2 \rangle - \langle\, k\, \rangle^2 = (n^2p^2 - np^2 + np) - n^2p^2 = np(1-p)$$

$$\boxed{\mu = np} \qquad \boxed{\sigma = \sqrt{np(1-p)} = \sqrt{npq}}$$

① Derivar respecto a p en la fórmula de potencia del binomio.

② Multiplicar por p y p^2 , y utilizar $q = 1-p$ para la binomial.

③ Remplazar $\langle\, k\, \rangle = \sum k\, P(k)$ y $\langle k^2 \rangle = \sum k^2 P(k)$, donde $P(k) = \binom{n}{k} p^{k-1} q^{n-k}$.